MATHEMATICS
UNIT TEST -1 KEY ANSWER

MARKING SCHEME

SECTION – A

Q.NO	OPTION	ANSWER
1	(1)	1
2	(3)	$k \neq -4$
3	(1)	$k^3 \det(A)$
4	(1)	is always consistent
5	(3)	$\lvert A\rvert^{n-1}$
6	(1)	$\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$
7	(3)	Infinitely many solution
8	(4)	$(A^{-1})^T$
9	(1)	1
10	(4)	no solution
11	(2)	Consistent and has unique solution
12	(4)	$\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$
13	(1)	B = 0
14	(1)	1
15	(3)	$\frac{1}{k} I$
16	(1)	$\Delta \neq 0$
17	(3)	1
18	(4)	Taking finite number of elementary transformations
19	(1)	1
20	(1)	1

SECTION B

Q.NO	CONTENT	MARKS
21	$AB = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$ $B^{-1} = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$ $(AB)^{-1} = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$ $B^{-1}A^{-1} = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$ $\therefore (AB)^{-1} = B^{-1}A^{-1}$	1 1 1 1 1 1
22	$\lvert A\rvert = \begin{vmatrix} 4 & -3 \\ 2 & 1 \end{vmatrix} = 10$ $adj\ A = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$ $A(adjA) = 10 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \lvert A\rvert I$ $(adjA)A = 10 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \lvert A\rvert I$ $\therefore A(adjA) = (adjA)A = \lvert A\rvert I$	1 1 2 1 1
23	$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{bmatrix}$ $\sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 6 & -16 \end{bmatrix}$ $R_2 \rightarrow R_2 - 2R_1$ $R_3 \rightarrow R_3 - 3R_1$ $\sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3 \rightarrow R_3 - 2R_2$ $\rho(A) = 2$	3 2 1
24	$\Delta = \begin{vmatrix} 2 & 2 & 1 \\ 1 & -1 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 0$ $\Delta_X = \begin{vmatrix} 5 & 2 & 1 \\ 1 & -1 & 1 \\ 4 & 1 & 2 \end{vmatrix} \neq 0$ Since $\Delta = 0$ and $\Delta_X \neq 0$, the system is inconsistent and has no solution	2 2 2
25	$\lvert A\rvert = \lvert A^T\rvert = 2$ $A^{-1} = \frac{1}{2}\begin{bmatrix} 5 & -3 \\ 4 & -2 \end{bmatrix}$ $(A^{-1})^T = \frac{1}{2}\begin{bmatrix} 5 & 4 \\ -3 & -2 \end{bmatrix}$ $adj(A^T) = \begin{bmatrix} 5 & 4 \\ -3 & -2 \end{bmatrix}$ $(A^T)^{-1} = \frac{1}{2}\begin{bmatrix} 5 & 4 \\ -3 & -2 \end{bmatrix}$ $(A^{-1})^T = (A^T)^{-1}$	1 1 1 1 1 1
26	$\lvert A\rvert = -1$ $[A_{ij}] = \begin{bmatrix} 1 & -4 & -4 \\ -2 & 3 & 4 \\ 2 & -4 & -5 \end{bmatrix}$ $adjA = \begin{bmatrix} 1 & -2 & 2 \\ -4 & 3 & -4 \\ -4 & 4 & -5 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ 4 & -4 & 5 \end{bmatrix} = A$	1 1+1+1 1 1
27(a)	$\Delta = 0$	1

	$\Delta_x = 0$ $\Delta_y = 0$ Since $\Delta = 0$, $\Delta_x = 0$ and $\Delta_y = 0$ and atleast one element is non-zero, it has infinitely many solutions y = k, $x = \frac{8-3k}{2}; k \in R$	1 1 1 1 1
27(b)	$\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 11 \end{bmatrix}$ $\lvert A \rvert = -1 \neq 0$ $A^{-1} = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$ $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 7 \\ 11 \end{bmatrix}$ $x = 3, y = -1$	1 1 2 * 1 1

SECTION – C

Q.NO	CONTENT	MARKS
28	$x + y + z = 100$ $12x + 13y + 15z = 1250$ $z = k \in R$ $x + y = 100 - k$ $12x + 13y = 1250 - 15k$ $\Delta = 1$ $\Delta_x = 50 + 2k$ $\Delta_y = 50 - 3k$ Solution set is $(50 + 2k, 50 - 3k, k)$; $0 \leq k \leq 16$ Taking k = 0, 1, 2 the solutions are (50,50,0), (52,47,1), (54,44,2)	1 1 2 2 2 2
29	$[A, B] = \begin{bmatrix} 2 & 5 & 7 & 52 \\ 1 & 1 & 1 & 9 \\ 2 & 1 & -1 & 0 \end{bmatrix}$ $\sim \begin{bmatrix} 1 & 1 & 1 & 9 \\ 2 & 5 & 7 & 52 \\ 2 & 1 & -1 & 0 \end{bmatrix}$ $R_1 \leftrightarrow R_2$ $\sim \begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & -1 & -3 & -18 \end{bmatrix}$ $R_2 \to R_2 - 2R_1$ $R_3 \to R_3 - 2R_1$ $\sim \begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & -1 & -3 & -18 \\ 0 & 3 & 5 & 34 \end{bmatrix}$ $R_2 \leftrightarrow R_3$ $\sim \begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & -1 & -3 & -18 \\ 0 & 0 & -4 & -20 \end{bmatrix}$ $R_3 \to R_3 + 3R_2$ $\rho(A) = \rho(A, B) = 3$ System has unique solution $\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -3 \\ 0 & 0 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -18 \\ -20 \end{bmatrix}$ $x + y + z = 9$ $-y - 3z = -18$ $-4z = -20$ Solution is $x = 1, y = 3, z = 5$	1 1 2 1 1 1 1 1 1
30	$[A, B] = \begin{bmatrix} 1 & 1 & 3 & 0 \\ 4 & 3 & \mu & 0 \\ 2 & 1 & 2 & 0 \end{bmatrix}$ $\sim \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & \mu - 12 & 0 \\ 0 & -1 & -4 & 0 \end{bmatrix}$ $R_2 \to R_2 - 4R_1$ $R_3 \to R_3 - 2R_1$ $\sim \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & \mu - 12 & 0 \\ 0 & 0 & 8 - \mu & 0 \end{bmatrix}$ $R_3 \to R_3 + R_2$ Case (i) $If\ \mu \neq 8$ $\rho(A) = \rho(A, B) = 3$ The system has trivial solution Case (ii) I $If\ \mu = 8$ $\rho(A) = \rho(A, B) = 2$ $< number\ of\ unknowns$ Given system is equivalent to $x + y + 3z = 0$ $y + 4z = 0$ Taking z = k, x = k and y = $-4k$; $k \in R - \{0\}$;	2 2 1 2 3
31	$[A, B] = \begin{bmatrix} k & 1 & 1 & 1 \\ 1 & k & 1 & 1 \\ 1 & 1 & k & 1 \end{bmatrix}$ $\sim \begin{bmatrix} 1 & 1 & k & 1 \\ 1 & k & 1 & 1 \\ k & 1 & 1 & 1 \end{bmatrix}$ $R_1 \leftrightarrow R_3$ $\sim \begin{bmatrix} 1 & 1 & k & 1 \\ 0 & k-1 & 1-k & 0 \\ 0 & 1-k & 1-k^2 & 1-k \end{bmatrix}$ $R_2 \to R_2 - R_1$ $R_3 \to R_3 - kR_1$	1 2

	$\sim\begin{bmatrix}1 & 1 & k\\ 0 & k-1 & 1-k\\ 0 & 0 & (2+k)(1-k)\end{bmatrix}$	1
	Case (i) k = 1 $\rho(A)=\rho(A,B)=1$ The system is consistent and has infinitely many solutions	2
	Case (ii) k = -2 $\rho(A,B)=3$ and $\rho(A)=2$ The system is inconsistent and has no solution	2
	Case (iii) $k\neq -2\,;k\neq -1$ $\rho(A)=\rho(A,B)=3$ $=\textit{the number of unknowns}$ System is consistent and has a unique solution	2
32	$\Delta=\begin{vmatrix}1 & 2 & 1\\ 2 & 4 & 2\\ 1 & -2 & -1\end{vmatrix}=0$	1
	$\Delta_x=\begin{vmatrix}2 & 2 & 1\\ 4 & 4 & 2\\ 0 & -2 & -1\end{vmatrix}=0$	1
	$\Delta_y=\begin{vmatrix}1 & 2 & 1\\ 2 & 4 & 2\\ 1 & 0 & -1\end{vmatrix}=0$	1
	$\Delta_z=\begin{vmatrix}1 & 2 & 2\\ 2 & 4 & 4\\ 1 & -2 & 0\end{vmatrix}=0$	1
	$\Delta=\Delta_x=\Delta_y=\Delta_z=0$ and one 2×2 minor of $\Delta\neq 0$	1
	Let z = k $x+2y=2-k$ $x-2y=k$	2
	$\Delta=-4\quad \Delta_x=-4$ $\Delta_y=2(k-1)$	1
	$\left(1,\frac{1-k}{2},k\right)$ $k\in R$is the solution	2
33	$\|A\|=1$	1
	$[A_{ij}]=\frac{1}{9}\begin{bmatrix}6 & 6 & 3\\ -6 & 3 & 6\\ 3 & -6 & 6\end{bmatrix}$	1+1 +1
	$adjA=\frac{1}{9}\begin{bmatrix}6 & -6 & 3\\ 6 & 3 & -6\\ 3 & 6 & 6\end{bmatrix}$	1
	$=\frac{1}{3}\begin{bmatrix}2 & -2 & 1\\ 2 & 1 & -2\\ 1 & 2 & 2\end{bmatrix}$	1

	$A^{-1}=\frac{1}{3}\begin{bmatrix}2 & -2 & 1\\ 2 & 1 & -2\\ 1 & 2 & 2\end{bmatrix}$	2* 1
	$A^{T}=\frac{1}{3}\begin{bmatrix}2 & -2 & 1\\ 2 & 1 & -2\\ 1 & 2 & 2\end{bmatrix}$	1
	$A^{-1}=A^{T}$	
34(a)	$[A,B]=\begin{bmatrix}1 & 1 & 1 & 2\\ 2 & 1 & -2 & 2\\ \lambda & 1 & 4 & 2\end{bmatrix}$	2
	$\sim\begin{bmatrix}1 & 1 & 1 & 2\\ 0 & -1 & -4 & -2\\ 0 & 1-\lambda & 4-\lambda & 2-2\lambda\end{bmatrix}$ $R_2\to R_2-2R_1$ $R_3\to R_3-\lambda R_1$	2
	$\sim\begin{bmatrix}1 & 1 & 1 & 2\\ 0 & -1 & -4 & -2\\ 0 & -\lambda & -\lambda & -2\lambda\end{bmatrix}$ $R_3\to R_3+R_2$	1
	$\sim\begin{bmatrix}1 & 1 & 1 & 2\\ 0 & -1 & -4 & -2\\ 0 & 0 & 3\lambda & 0\end{bmatrix}$ $R_3\to R_3-\lambda R_2$	1
	Case (i) $\lambda=0$ $\rho(A)=\rho(A,B)=2<3$ System is consistent and has infinitely many solutions	2
	Case (ii) $\lambda\neq 0$ $\rho(A)=\rho(A,B)=3$ The system is consistent and has unique solution	2
34(b)	$a+2b-c=1$ $2a+4b+c=5$ $3a-2b-2c=0$	2
	$\Delta=24$	1
	$\Delta_a=24$	1
	$\Delta_b=12$	1
	$\Delta_c=24$	1
	$a=1\,;b=\frac{1}{2}\,;c=1$	2
	Solution is $x=1,y=2,z=1$	2